Università di Bologna – Campus di Rimini – Corso di laurea in Farmacia e CQPS

Esame MATEMATICA 17/06/2016 – Docente: Stefano Bordoni

STUDENTE: ______________________; CORSO di LAUREA: __________

MATRICOLA: _______; N° documento: ___________; FIRMA: ___________

1. Calcolare: $\log_k(k/\sqrt[2]{k})$, $\dfrac{(n+1)!-n!}{n!}$ [2]

2. Risolvere: $(x-2)^4>0$; $(x-2)^3\le 0$; $\dfrac{1}{(x-2)^3}\ge 0$; $\sqrt[2]{(x-2)^3}\le 0$; $3^{1-x}\ge\dfrac{1}{3}$;

$|3x-x^2|\le 2$; $\sqrt[2]{1-2x^2}\le 2$; $\log_{\frac{1}{3}}(x^2-1)>-1$ [11]

3. Calcolare la funzione derivata e una funzione primitiva della funzione $y=\sqrt[3]{x^2}$ [3]

4. Determinare dominio, grafico e codominio (studio globale) della funzione $y=e^x-1$, ed eventuali estremi superiore e inferiore. Controllare se la funzione è pari o dispari, e determinare inoltre la funzione derivata $f'(x)$. [5]

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5. Eseguire lo studio analitico della funzione $y=\dfrac{x^2-1}{x}$. [6]

6. In una specie di ratti, la probabilità che nasca una femmina è 7/12. Indicare la probabilità che su 4 figli, siano metà maschi e metà femmine. [3]

PER LA LODE:

Dimostrare che l'insieme Q è numerabile.

VOTO: ______

MATEMATICA 17/6/2016

1. • $K^y = \frac{K}{\sqrt[2]{K}}$ $\quad K^y = \sqrt[2]{K}$ $\quad K^y = K^{1/2}$ $\quad y = \frac{1}{2}$

• $\frac{(n+1)\cdot n! - n!}{n!} = \frac{\cancel{n!}\left[(n+1)-1\right]}{\cancel{n!}} = n$

2. • $(x-2)^4 > 0$ $\quad x-2 \neq 0$ $\quad x \neq 2$ $\quad x \in \mathbb{R} \setminus \{2\}$

• $(x-2)^3 \leq 0$ $\quad x-2 \leq 0$ $\quad x \leq 2$ $\quad x \in \left]-\infty; 2\right]$

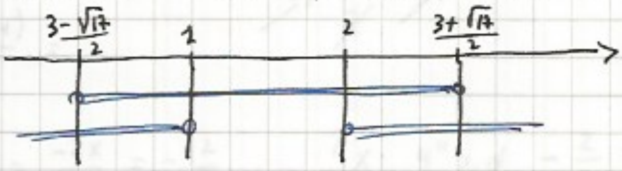
• $\frac{1}{(x-2)^3} \geq 0$ $\quad x-2 > 0$ $\quad x > 2$ $\quad x \in \left]2; +\infty\right[$

• $\sqrt{(x-2)^3} \leq 0$ $\quad$ Deve essere $x-2 \geq 0 \rightarrow$ $\quad x = 2$ $\quad x \in \{2\}$

• $3^{1-x} \geq 3^{-1}$ $\quad 1-x \geq -1$ $\quad -x \geq -2$ $\quad x \leq 2$ $\quad x \in \left]-\infty; 2\right]$

• $|3x - x^2| \leq 2$ $\quad -2 \leq 3x - x^2 \leq 2$ $\quad \begin{cases} 3x - x^2 \geq -2 \\ 3x - x^2 \leq 2 \end{cases}$ $\quad \begin{cases} -x^2 + 3x + 2 \geq 0 \\ -x^2 + 3x - 2 \leq 0 \end{cases}$

$\begin{cases} x^2 - 3x - 2 \leq 0 \\ x^2 - 3x + 2 \geq 0 \end{cases}$ $\quad \begin{cases} \frac{3-\sqrt{17}}{2} \leq x \leq \frac{3+\sqrt{17}}{2} \\ x \leq 1 \ \vee \ x \geq 2 \end{cases}$

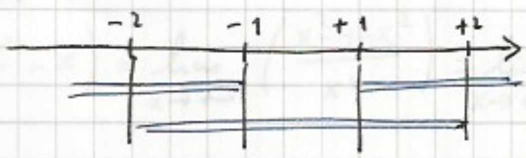


$x \in \left[\frac{3-\sqrt{17}}{2}; 1\right] \cup \left[2; \frac{3+\sqrt{17}}{2}\right]$

• $\sqrt{1-2x^2} \leq 2$ $\quad \begin{cases} 1-2x^2 \geq 0 \\ 1-2x^2 \leq 4 \end{cases}$ $\quad \begin{cases} 2x^2 - 1 \leq 0 \\ -2x^2 - 3 \leq 0 \end{cases}$ $\quad \begin{cases} -\sqrt{\frac{1}{2}} \leq x \leq +\sqrt{\frac{1}{2}} \\ 2x^2 + 3 \geq 0 \end{cases}$ $\quad \begin{cases} x \in \left[-\frac{\sqrt{2}}{2}; +\frac{\sqrt{2}}{2}\right] \\ \forall x \in \mathbb{R} \end{cases}$

$\Rightarrow x \in \left[-\frac{\sqrt{2}}{2}; +\frac{\sqrt{2}}{2}\right]$

• $\log_{1/3}(x^2-1) > -1$ $\quad \begin{cases} x^2 - 1 > 0 \\ x^2 - 1 < \left(\frac{1}{3}\right)^{-1} \end{cases}$ $\quad \begin{cases} x < -1 \ \vee \ x > +1 \\ x^2 - 1 < 3 \end{cases}$ $\quad \begin{cases} \\ x^2 - 4 < 0 \end{cases}$ $\quad \begin{cases} x < -1 \ \vee \ x > +1 \\ -2 < x < +2 \end{cases}$



$x \in \left]-2; -1\right[\cup \left]+1; +2\right[$

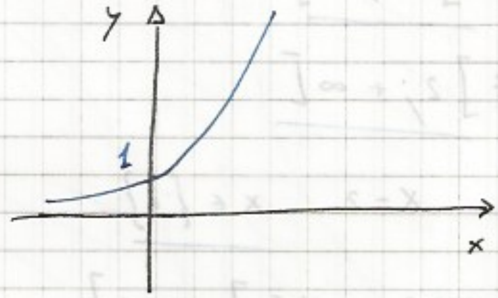
• 3. $y = \sqrt[3]{x^2} = x^{2/3}$

$y' = \frac{2}{3} x^{\frac{2}{3}-1} = \frac{2}{3} x^{-1/3} = \frac{2}{3} \frac{1}{x^{1/3}} = \frac{2}{3} \cdot \frac{1}{\sqrt[3]{x}}$

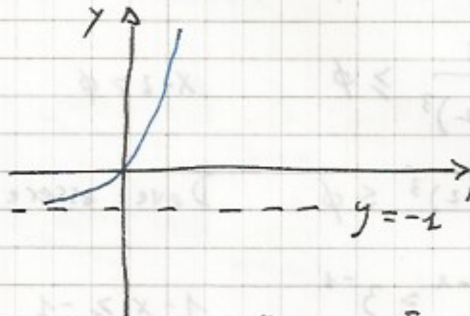
$\phi(x) = \frac{x^{2/3+1}}{\frac{2}{3}+1} = \frac{x^{5/3}}{\frac{5}{3}} = \frac{3}{5} \cdot \sqrt[3]{x^5}$

4. $y = e^x \xrightarrow[1\,D]{T_y} y = e^x - 1$

$y + 1 = e^x$



$\xrightarrow{T_y}$



D: $\mathbb{R}$; Cd: $]-1\,;+\infty[$

inf $= -1$; sup $\nexists$ $(+\infty)$

$f(-x) = e^{-x} - 1$ → $\neq f(x)$ NO PARI
→ $\neq -f(-x)$ NO DISPARI

$f'(x) = e^x$

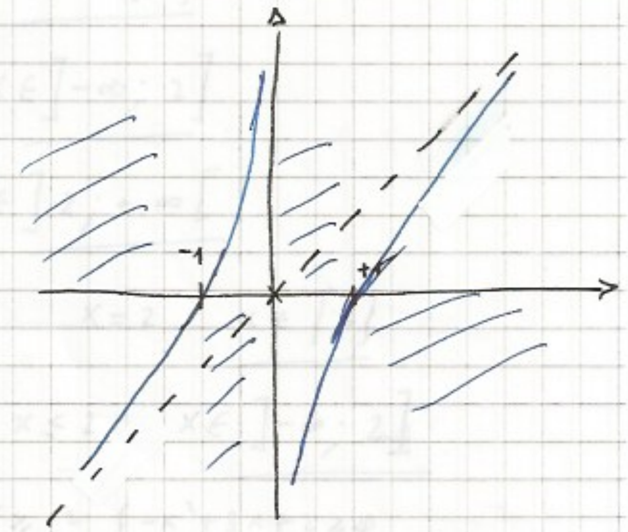
5. $y = \dfrac{x^2-1}{x}$ $\quad D: \mathbb{R} \setminus \{0\}$

- $f(-x) = \dfrac{x^2-1}{-x} = -\dfrac{x^2-1}{x} = -f(x)$ DISPARI

- $? \, x: \dfrac{x^2-1}{x} \geq 0 \quad \nearrow x^2-1 \geq 0 \quad x \leq -1 \vee x \geq +1$
  $\searrow x \geq 0$

- $\lim\limits_{x \to 0^+} \dfrac{x^2-1}{x} = \dfrac{-1}{0^+} = -\infty$

- $\lim\limits_{x \to +\infty} \dfrac{x^2-1}{x} \sim \lim\limits_{x \to +\infty} \dfrac{x^2}{x} = \lim\limits_{x \to +\infty} x = +\infty$

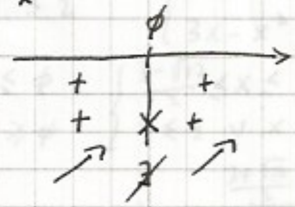


Per simmetria: $\lim\limits_{x \to 0^-} f(x) = +\infty$

$\lim\limits_{x \to -\infty} f(x) = -\infty$

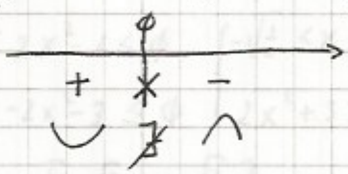
- $y' = \dfrac{2x \cdot x - 1 \cdot (x^2-1)}{x^2} = \dfrac{2x^2 - x^2 + 1}{x^2} = \dfrac{x^2+1}{x^2}$

$y' > 0 \quad \forall x \in \mathbb{R} \setminus \{0\}$



- $y'' = \dfrac{2x \cdot x^2 - 2x \cdot (x^2+1)}{x^4} =$

$= \dfrac{2x^3 - 2x^3 - 2x}{x^4} = \dfrac{-2x}{x^4} = -\dfrac{2}{x^3}$ $\quad ? \, x: y'' \geq 0 \quad -\dfrac{2}{x^3} \geq 0 \quad \dfrac{2}{x^3} \leq 0 \quad x^3 < 0 \quad x < 0$



- Ricerca eventuale asintoto obliquo: $m = \lim\limits_{x \to +\infty} \dfrac{x^2-1}{x} \cdot \dfrac{1}{x} \sim \lim\limits_{x \to +\infty} \dfrac{x^2}{x^2} = +1$

Anche $\lim\limits_{x \to -\infty} \dfrac{f(x)}{x} = +1$

$q = \lim\limits_{x \to +\infty} \left(\dfrac{x^2-1}{x} - x \right) = \lim\limits_{x \to +\infty} \left(\dfrac{x^2-1-x^2}{x^2} \right) = \lim\limits_{x \to +\infty} \left(\dfrac{-1}{x^2} \right) = 0 = \lim\limits_{x \to -\infty} \left(-\dfrac{1}{x^2} \right)$

$\Rightarrow \exists$ asintoto obliquo $y = x$

6. $p(F) = \frac{7}{12}$ $\quad p(M) = 1 - \frac{7}{12} = \frac{5}{15}$

$$p(2M/4) = \binom{4}{2} \cdot \left(\frac{7}{12}\right)^2 \cdot \left(\frac{5}{12}\right)^2 = \frac{4!}{2! \cdot 2!} \frac{7^2}{12^2} \frac{5^2}{12^2} = \frac{\cancel{4} \cdot 3 \cdot 2}{\cancel{2 \cdot 2}} \frac{7^2 \cdot 5^2}{12^4}$$

$$= 6 \cdot \frac{7^2 \cdot 5^2}{12^4}$$